A Low-Complexity PTS Scheme with the Hybrid Subblock Partition Method for PAPR Reduction in OFDM Systems

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SUMMARY The technique of partial transmit sequences (PTS) is effective in reducing the peak-to-average power ratio (PAPR) of orthogonal frequency division multiplexing (OFDM) signals. However, the conventional PTS (CPTS) scheme has high computation complexity because it needs several inverse fast Fourier transform (IFFT) units and an optimization process to find the candidate signal with the lowest PAPR. In this paper, we propose a new low-complexity PTS scheme for OFDM systems, in which a hybrid subblock partition method (SPM) is used to reduce the complexity that results from the IFFT computations and the optimization process. Also, the PAPR reduction performance of the proposed PTS scheme is further enhanced by multiplying a selected subblock with a predefined phase rotation vector to form a new subblock. The time-domain signal of the new subblock can be obtained simply by performing a circularly-shift-left operation on the IFFT output of the selected subblock. Computer simulations show that the proposed PTS scheme achieves a PAPR reduction performance close to that of the CPTS scheme with the pseudo-random SPM, but with much lower computation complexity.

Keywords: orthogonal frequency division multiplexing (OFDM), partial transmit sequences (PTS), peak-to-average power ratio (PAPR)

1. Introduction

Orthogonal frequency division multiplexing (OFDM) is a promising technique for high-data-rate wireless communications because it is robust to multipath fading and has high spectral utilization efficiency [1]. OFDM is widely used in various modern wireless communications systems, such as wireless local area networks (WLANs) [2], wireless metropolitan area networks (WMANs) [3], and 3GPP Long Term Evolution (LTE)/LTE-Advanced (LTE-A) systems [4]. However, the transmitted signal of OFDM systems may have a high peak-to-average power ratio (PAPR) due to the nature of multicarrier communications. When a high-PAPR OFDM signal is passed through nonlinear devices (e.g., high-power amplifiers), it may cause undesired signal distortion and degrade system performance.

Numerous methods have been presented to reduce the PAPR of OFDM systems [5], including clipping [6], [7], companding transform [8], [9], selected mapping (SLM) [10]–[13], and partial transmit sequences (PTS) [14]–[21]. Clipping and companding transform techniques deal with the discrete-time OFDM signals of the inverse fast Fourier transform (IFFT) output. These two methods have low complexity, but they are nonlinear processes and may cause distortion of the transmitted signals.

SLM and PTS schemes are symbol-scrambling techniques, where a set of candidate signals are formed from which to select the signal to be transmitted. In the conventional PTS (CPTS) scheme, the input data block is partitioned into several disjointed subblocks evenly by using a predefined subblock partition method (SPM), which generally includes pseudo-random, adjacent, and interleaving processes. Then, the IFFT outputs of the subblocks are multiplied by a set of rotation factors and added to form various candidate signals; the signal with the lowest PAPR value is selected for transmission. The CPTS scheme is a non-distortion technique and has good PAPR reduction performance, but it has high computation complexity and needs to send side information to the receiver for recovering the received signals.

Some methods have been proposed to eliminate the need for side information for the CPTS scheme [19]–[21]. Also, many methods have been presented to simplify the complexity of the CPTS scheme, such as reducing the number of candidate signals generated [15], reducing the number of samples used to estimate the peak power of candidate signals [16], [17], and reducing the computation complexity of forming candidate signals [18]. However, these methods, [15]–[18], do not simplify the IFFT complexity.

In this paper, we restrict our attention to reducing the complexity that results from the IFFT computations and the optimization process for the PTS technique. In the proposed PTS scheme, a hybrid SPM that combines pseudo-random and interleaving methods is used to partition the input data block into \(M\) subblocks evenly. Furthermore, one subblock is selected and multiplied with a predetermined phase rotation vector to form a new subblock for generating more candidate signals, where the corresponding IFFT output of the new subblock can be obtained easily by circularly-shifting-left the IFFT output of the selected subblock. Computer simulations show that the proposed hybrid PTS scheme can achieve a PAPR reduction performance close to that of the CPTS scheme with the pseudo-random SPM, but with much lower complexity, both in the IFFT computations and in the optimization process. In addition, the proposed PTS scheme can be used to further reduce the complexity of the work in...
PAPR of the discrete-time signal (1) can be realized easily by using an 
systems, the CPTS scheme, and the reduced-complexity 
PTS (RC-PTS) scheme of [17]. In Sect. 3, a new PTS 
scheme with a hybrid SPM is proposed, where the complexity 
that results from the IFFTs and the optimization process 
is reduced. In Sect. 4, a new algorithm is developed to en-
chance the PAPR reduction performance of the proposed PTS 
scheme. In Sect. 5, the computation complexities of the pro-
posed PTS schemes and of two related works are analyzed.
In Sect. 6, computer simulations are presented to compare 
the PAPR reduction performance and the computation com-
bility of finding the peak correctly by using 
the sample with time index

In OFDM systems with non-zero elements such that

PAPR of the discrete-time signal $x = \{x_0, x_1, \ldots, x_{N-1}\}$ is defined as

$$\text{PAPR} = \frac{\max_{0\leq n \leq N-1} |x_n|^2}{E[|x|^2]}$$

where $E[\cdot]$ denotes the expectation operator.

2.2 Conventional PTS (CPTS) Scheme

In the CPTS scheme, the input data block $X$ is first partitioned into $M$ disjointed subblocks by using a predefined 
SPM [14]. The $m$-th partitioned subblock can be expressed as

$$X_m = [X_{m,0}, X_{m,1}, \ldots, X_{m,N-1}]^T, 1 \leq m \leq M$$

and each subblock has only $N/M$ nonzero elements such that

$$X = \sum_{m=1}^{M} X_m. \tag{4}$$

The IFFT of subblock $X_m$ (i.e., $x_m = IFFT[X_m] = \{x_{m,0}, x_{m,1}, \ldots, x_{m,N-1}\}$) is multiplied by a set of rotation 
factors $b_m^c = e^{i\theta_m}$ with $\theta_m \in [0, 2\pi]$ and then combined to form the $c$-th candidate signal $x^c$, expressed as

$$x^c = \sum_{m=1}^{M} b_m^c x_m = \{x_0^c, x_1^c, \ldots, x_{N-1}^c\}, 1 \leq c \leq C \tag{5}$$

where $C$ is the number of candidate signals. The optimal candidate signal $x^{copt}$ with the lowest PAPR among the $C$ candidate signals is transmitted. Let $b^c = [b_1^c, b_2^c, \ldots, b_M^c]^T$ be the vector of rotation factors used to generate $x^c$. In general, $b_i^c$ is fixed without degrading the PAPR reduction performance of the CPTS scheme. The CPTS scheme has good PAPR reduction performance, but it has high computation complexity to compute the IFFT of $M$ subblocks and to find the optimal output signal $x^{copt}$. Also, it must send side information to the receiver to indicate the optimal vector $b^{copt} = [b_1^{copt}, b_2^{copt}, \ldots, b_M^{copt}]^T$ used by the transmitted signal.

2.3 Reduced-Complexity PTS (RC-PTS) Scheme

A reduced-complexity PTS (RC-PTS) scheme for OFDM systems was proposed in [17], where the cost function of the sample with time index $n$ is defined as

$$Q_n = \sum_{m=1}^{M} |x_{m,n}|^2, n = 0, 1, \ldots, N-1 \tag{6}$$

where $M$ is the number of subblocks. During the optimization process, only the samples with $Q_n$ greater than or equal to the threshold $\alpha = \Phi^c/T$ are used to estimate the peak power of each candidate signal, in which $\Phi^c_n$ is defined as

$$\Phi^c_n = -\sigma^2 \ln \frac{\ln(1 - \gamma)}{-N(\frac{\pi}{3} \ln N)^{0.5}} \tag{7}$$

$\sigma^2$ is the mean power, and $\gamma$ is the lower bound of the prob-
ability of finding the peak correctly by using $Q_n$. The RC-
PTS scheme efficiently decreases the computation complex-
ity of the optimization process of the CPTS scheme, but it does not simplify the IFFT computations of $M$ subblocks. In the next section, we develop a new PTS method that simplifies the computation complexity of both the IFFTs and the optimization process.

3. New Low-Complexity PTS Scheme with a Hybrid 
SPM

The SPMs used in the PTS technique usually are divided into three categories: interleaving, adjacent, and pseudo-
random. Among the three SPMs, the interleaving SPM has the lowest computation complexity and the pseudo-random 
SPM has the best PAPR reduction performance. To trade off the PAPR reduction performance and the computation complex-
ity, we use a new hybrid SPM that combines the pseudo-random and interleaving methods in the proposed PTS scheme. We first use the pseudo-random SPM to di-
vide the input data block $X$ into $M_p$ disjointed subblocks even-
ly, denoted as

$$X_{mp} = [X_{mp,0}, X_{mp,1}, \ldots, X_{mp,N-1}]^T, 1 \leq m_p \leq M_p \tag{8}$$

Then, $X_{mp}$ is further divided into two disjointed subblocks
by using the interleaving SPM, expressed as

\[
\begin{align*}
X_{mp}^{(1)} &= \left[ X_{mp,0}^{(1)}, X_{mp,1}^{(1)}, \ldots, X_{mp,N-1}^{(1)} \right]^T \\
X_{mp}^{(2)} &= \left[ X_{mp,0}^{(2)}, X_{mp,1}^{(2)}, \ldots, X_{mp,N-1}^{(2)} \right]^T
\end{align*}
\]

(9a)

where there are only \(N/2M_p\) nonzero elements in subblock \(X_{mp}^{(i)}\), for \(i = 1, 2\), and \(1 \leq m_p \leq M_p\). The \(c\)-th candidate signals \(x'\) formed by \(X_{mp}^{(i)}\) of (9) are expressed as

\[
x' = 2 \sum_{i=1}^{M_p} \sum_{m_p=1}^{M_p} b_{mp}^{(i)c} IFFT \begin{bmatrix} X_{mp}^{(i)} \end{bmatrix}
\]

(10)

where \(X_{mp}^{(i)} = IFFT(X_{mp}^{(i)}) = [X_{mp,0}^{(i)}, X_{mp,1}^{(i)}, \ldots, X_{mp,N-1}^{(i)}]^T\) with elements expressed as

\[
x_{mp,n}^{(i)} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_{mp,k}^{(i)} e^{\frac{j2\pi kn}{N}}, i = 1, 2, n = 0, 1, \ldots, N - 1
\]

and \(b_{mp}^{(i)c}\) is the rotation factor for \(X_{mp}^{(i)}\).

In addition, there are \(N/2\) zeros interleaved in subblock \(X_{mp}^{(i)}\) for \(i = 1, 2\). The discrete-time signal \(x_{mp,n}^{(1)}\) of subblock \(X_{mp}^{(1)}\) of (9a) can be rearranged as

\[
x_{mp,n}^{(1)} = \frac{1}{\sqrt{N'}} \sum_{k=0}^{N'-1} X_{mp,k'} e^{\frac{j2\pi kn}{N'}} = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{N'}} \sum_{k'=0}^{N'-1} x_{mp,k'} e^{\frac{j2\pi kn}{N'}}
\]

(11)

where \(N' = N/2\) and \(k' = k/2\). It is clear that \(x_{mp,n}^{(1)}\) can be obtained by using an \(N/2\)-point IFFT computation. Also, \(x_{mp,n}^{(1)} = x_{mp,n+N'}\) because \(e^{\frac{j2\pi kn}{N'}} = e^{\frac{j2\pi (n+N')}{N'}}\) for \(n = 0, 1, \ldots, N' - 1\). \(x_{mp}^{(1)}\) has the format

\[
x_{mp}^{(1)} = \begin{bmatrix} x_{mp,1}^{(1)} \\ x_{mp,1}^{(1)} \end{bmatrix}^T \quad \text{for} \quad 1 \leq m_p \leq M_p
\]

(13)

where \(x_{mp}^{(1)}\) is the IFFT of the \(N/2\) elements \(X_{mp,k}\) of \(X_{mp}^{(1)}\) of (9a) with \(k = 2k'\) for \(k' = 0, 1, \ldots, N/2 - 1\). Similarly, \(x_{mp}^{(2)}\) has the format

\[
x_{mp}^{(2)} = \begin{bmatrix} x_{mp,1}^{(2)} \\ -x_{mp,1}^{(2)} \end{bmatrix}^T \quad \text{for} \quad 1 \leq m_p \leq M_p
\]

(14)

where \(x_{mp}^{(2)}\) is the IFFT of the \(N/2\) elements \(X_{mp,k}\) of \(X_{mp}^{(2)}\) of (9b) with \(k = 2k' + 1\) for \(k' = 0, 1, \ldots, N/2 - 1\). Therefore, the proposed hybrid PTS scheme needs only one \(N/2\)-point IFFT to acquire the time-domain signal \(x_{mp}^{(i)}\) for each subblock \(X_{mp}^{(i)}\) of (9), which clearly simplifies the IFFT computations of the CPTS scheme.

Also, from (13) and (14), \(x'\) in (10) can be rearranged

\[
x' = 2 \sum_{i=1}^{M_p} \sum_{m_p=1}^{M_p} b_{mp}^{(i)c} \begin{bmatrix} X_{mp}^{(i)} \end{bmatrix}
\]

(15)

We find that the two summation terms of the first half of (15) can be reused in the second half of (15). Thus, the number of complex additions required to form \(x'\) in (15) is reduced from \(N(2M_p - 1)\) to \(NM_p\), and the number of multiplications of phase factors is reduced from \(2NM_p\) to \(NM_p\).

4. Enhancement of the PAPR Reduction Performance for the Proposed PTS Scheme

4.1 Redundant Candidate Signals with the Same PAPR

Although the similarity of the elements of \(X_{mp}^{(i)}\) can be used to simplify the complexity of forming candidate signals in (15), it causes some of the candidate signals to have the same PAPR value. For example, if the rotation factors of two candidate signals \(x'^1\) and \(x'^2\) have the relationship \(b_{mp}^{(1)c} = b_{mp}^{(2)c}\) and \(b_{mp}^{(1)c} = -b_{mp}^{(2)c}\) for \(1 \leq m_p \leq M_p\), then \(x'^1\) and \(x'^2\) can be expressed as

\[
x'^1 = \sum_{m_p=1}^{M_p} b_{mp}^{(1)c} x_{mp}^{(1)} + \sum_{m_p=1}^{M_p} b_{mp}^{(2)c} x_{mp}^{(2)}
\]

(16a)

\[
x'^2 = \sum_{m_p=1}^{M_p} b_{mp}^{(1)c} x_{mp}^{(1)} - \sum_{m_p=1}^{M_p} b_{mp}^{(2)c} x_{mp}^{(2)}
\]

(16b)

It is clear that \(x'^2\) of (16b) is a circularly-shift-left version of \(x'^1\) of (16a) by \(N/2\) elements, so \(x'^1\) and \(x'^2\) have the same PAPR. In the case of \(M_p = 2\) and \(b_{mp}^{(i)c} \in \{±1, ±j\}\) for \(i = 1, 2\), \(1 \leq m_p \leq M_p\), there are 32 various PAPR values among the
64 candidate signals of the proposed PTS scheme, which is only one-half the number of the CPTS scheme using the pseudo-random or the adjacent SPM. These redundant candidate signals are invalid for the PAPR reduction performance of the proposed PTS scheme.

4.2 Increment of Valid Candidate Signals

To increase the number of valid candidate signals for the proposed PTS scheme, we multiply one of the subblocks \(X_{mp}^{(i)}\) of (9) by a phase rotation vector \(V = [V_0, V_1, V_2, \ldots, V_{N-1}]^T\) to form alternative frequency-domain data subblock \(\hat{X}_{mp}^{(i)}\), expressed as

\[
\hat{X}_{mp}^{(i)} = \begin{bmatrix}
\hat{x}_{mp, 0}^{(i)}, \hat{x}_{mp, 1}^{(i)}, \ldots, \hat{x}_{mp, N-1}^{(i)}
\end{bmatrix}^T = [V_0 \hat{x}_{mp, 0}^{(i)}, V_1 \hat{x}_{mp, 1}^{(i)}, \ldots, V_{N-1} \hat{x}_{mp, N-1}^{(i)}]^T. (17)
\]

Then, the corresponding IFFT output \(\hat{x}_{mp}^{(i)} = IFFT(\hat{X}_{mp}^{(i)}) = [\hat{x}_{mp, 0}^{(i)}, \hat{x}_{mp, 1}^{(i)}, \ldots, \hat{x}_{mp, N-1}^{(i)}]^{T}\) is used in (15) to form valid candidate signals. Let \(v = IFFT(V)\). The relation between \(\hat{x}_{mp}^{(i)}\) and \(x_{mp}^{(i)}\) can be expressed by

\[
x_{mp}^{(i)} = TVx_{mp}^{(i)}, \quad i = 0, 1, 2
\]

in which

\[
TV = [v, v^{(1)}, v^{(2)}, \ldots, v^{(N-1)}]
\]

is referred to as the conversion matrix along with the vector \(v\), and \(v^{(k)}\) is a circularly-shift-down version of the column vector \(v\) by \(k\) elements [11].

Some low-complexity \(TV\) proposed in [11], [12] can be used to simplify the multiplication of (18). For example, in OFDM systems with \(N = 16\) subcarriers, if \(V_1 = [V_B, V_B]^T = [1, 1, -j, -1, 1, j, -1, -1, 1, 1, j, -1, -1, -1, 1, -j]^{T}\) with the basic vector \(V_0 = [1, 1, -j, -1, -1, 1, j, -1, -1, -1, -1, 1, -j, 1, j, -1]^{T}\), then the first column vector of \(TV\), \(v_1 = IFFT(V_1) = 0.5 \times [0, 0, 1, 0, 1, 1, 1, 0, 0, 0, 0, 0, 1, 0, -j, 0, 0, 0]^{T}\). The first element of \(\hat{x}_{mp}^{(i)}, g_{mp, 0}\) can be obtained by

\[
\hat{x}_{mp, 0}^{(i)} = v_1^T x_{mp}^{(i)} = 0.5 \left(\hat{x}_{mp, 2}^{(i)} + j x_{mp, 10}^{(i)} - j \hat{x}_{mp, 12}^{(i)}\right). (20)
\]

Using the similarity of the elements in \(x_{mp}^{(i)}\), described in (13) and (14), (20) can be rearranged as

\[
x_{mp, 0}^{(i)} = \begin{cases} 
\hat{x}_{mp, 2}^{(i)}, & \text{if } i = 1 \\
j \hat{x}_{mp, 10}^{(i)} & \text{if } i = 2.
\end{cases} (21)
\]

From (21), we can obtain \(x_{mp}^{(i)}\) by circularly-shifting-left the elements of \(x_{mp}^{(i)}\) by two (i.e., \(N/8\)) elements if \(i = 1\), or by multiplying \(j\) and circularly-shifting-left the elements of \(x_{mp}^{(i)}\) by four (i.e., \(N/4\)) elements if \(i = 2\). In the following, we choose \(x_{mp}^{(i=1)}\) and \(V = [V_B, V_B, \ldots, V_B]^{T}\) with the basic vector \(V_0 = [1, 1, -j, -1, 1, j, -1, -1, -1, -1, -1, -1, 1, j, -1, 1]^{T}\) in (17) such that \(x_{mp}^{(i=1)}\) can be obtained by circularly-shifting-left the elements of \(x_{mp}^{(i)}\) by \(N/8\) elements.

4.3 Cost Functions for the Proposed PTS Scheme

The proposed PTS scheme also can reduce the complexity of the RC-PTS scheme in [17]. We define the cost function of the proposed PTS scheme by

\[
Q_p^n = \sum_{m_p=1}^{M_p} \left| x_{mp,n}^{(1)} \right|^2 + \left| x_{mp,n}^{(2)} \right|^2, \quad n = 0, 1, \ldots, N - 1. (22)
\]

Only the samples with \(Q_p^n\) greater than a defined threshold value \(\alpha\) are used to estimate the peak power of candidate signals in the optimization process. Due to the similarity of the elements in \(x_{mp}^{(1)}\) and \(x_{mp}^{(2)}\) shown in (13) and (14), \(Q_p^n\) of (22) have the relation

\[
Q_p^n = Q_p^{N/2+n} \quad \text{for} \quad N = 0, 1, \ldots, N/2 - 1. (23)
\]

Therefore, only \(Q_p^n\) with \(n = 0, 1, \ldots, N/2 - 1\) are generated by (22) and are compared with the threshold in the proposed PTS scheme, which is only one-half the number in the RC-PTS scheme of [17].

In addition, another set of the cost function \(\hat{Q}_p^n\) is formed when \(\hat{x}_{mp}^{(i=1)}\) is used in (15) to generate more candidate signals. The difference between \(Q_p^n\) and \(\hat{Q}_p^n\) is the terms \(\hat{x}_{mp, 1}^{(i)}\) and \(x_{mp, 1}^{(i)}\) used in (22); the other terms are the same. Also, we know that \(|\hat{x}_{mp, 1}^{(i)}|^2 = |x_{mp, 1}^{(i)}|^2\) from the derivation of (21). Therefore, to generate \(\hat{Q}_p^n\) with \(n = 0, 1, \ldots, N/2 - 1\) needs only \(N/2\) real additions if the intermediate results of computing \(\hat{Q}_p^n\) are reused. Figure 1 shows the block diagram of the proposed PTS scheme with \(M_p = 2\), where the cost function is included and the CSL block represents the circularly-shifting-left operation.

5. Computation Complexity Analysis

Assume that, in OFDM systems with \(N\) subcarriers, the input data block \(X\) is divided into \(M\) disjointed subblocks using the pseudo-random SPM in the CPTS scheme. Then, \(M\) \(N\)-point IFFT units are used to generate the discrete-time domain signals of \(M\) subblocks, which need \(0.5MN\log_2 N\) complex multiplications and \(MN\log_2 N\) complex additions. Let the rotation factor \(b_1^n\) be 1. The CPTS scheme with \(C\)
candidate signals needs \( CN(M - 1) \) multiplications of rotation factors and \( CN(M - 1) \) complex additions in (5); \( CN \) complex multiplications to compute the sample powers; and \( CN - 1 \) comparisons to find the optimal signal among the \( C \) candidate signals in the optimization process.

In the proposed PTS scheme with \( M = 2M_p \) subblocks, \( N \) \( N/2 \)-point IFFT units are used in (12), which needs \( 0.25MN \log_2 N/2 \) complex multiplications and \( 0.5MN \log_2 N/2 \) complex additions. Two sets of cost functions, \( Q_n^p \) and \( \tilde{Q}_n^p \), are formed to select samples for peak power estimation. In generating \( Q_n^p \) with \( n = 0, 1, \ldots, N/2 - 1 \) needs \( MN/2 \) complex multiplications and \( (M - 1)N/2 \) real additions in (22). Also, generating \( \tilde{Q}_n^p \) with \( n = 0, 1, \ldots, N/2 - 1 \) needs only \( N/2 \) real additions if the intermediate results from computing \( Q_n^p \) are reused. During the optimization process, the proposed PTS scheme uses only the samples with cost functions greater than or equal to a threshold \( \alpha \) to estimate the peak power. Let \( p_n \) be the probability that the cost functions \( Q_n^p \) or \( \tilde{Q}_n^p \) are greater than \( \alpha \) for \( n = 0, 1, \ldots, N - 1 \). Then, \( p_nN \) samples are used to estimate the peak power of each candidate signal in the proposed PTS schemes. The probability density function of \( p_n \) was derived in [17].

Let \( C_1 \) and \( C_2 \) be the number of candidate signals generated by using \( x_1^{(1)} \) and \( x_1^{(1)} \) in (15), respectively. Then, generating candidate signals \( x^{(1)} \) with \( l = e \leq C_1 \) needs \( C_1(M - 1)p_nN/2 \) multiplications of rotation factors and \( C_1Mpn/2 \) complex additions in (15). In addition, when we use \( x_1^{(1)} \) to replace \( x_k^{(1)} \) to generate another set of candidate signals \( x^{(1)} \) with \( l = e \leq C_2 \), the intermediate results of generating \( x^{(1)} \) are reused. Therefore, to generate \( x^{(1)} \) with \( l = e \leq C_2 \) needs \( C_2Mpn/2 \) complex additions to compute the first summation term in (15) and \( C_2Mp_nN/2 \) complex additions to find the sum/difference of the first and second summation terms in (15). Finally, \( (C_1 + C_2)p_nN/2 \) complex multiplications are required to compute the sample powers and \( (C_1 + C_2)p_nN - 1 \) comparison operations are required to find the optimal signal among \( C_1 + C_2 \) candidate signals. In the following, we choose \( C_1 = C_2 = C/2 \). Table 1 illustrates the comparisons of computation complexity between the CPTS scheme, the RC-PTS scheme in [17], and the proposed PTS scheme, where the \( L \) times oversampling technique is used to generate the discrete-time domain signals so that the terms \( N \) are replaced by \( LN \). Note that a complex addition is equivalent to two real additions and a comparison is equivalent to a real addition.

### 6. Simulation Results

Computer simulations are used to compare the PAPR reduction performance for the proposed PTS scheme, the CPTS scheme, and the RC-PTS scheme in [17]. The OFDM system is assumed to have \( N = 64 \) subcarriers with 16-QAM modulation and the mean power is 1. The \( L = 4 \) times oversampling technique is used to obtain the discrete-time OFDM signals. The CPTS and the RC-PTS schemes use the pseudo-random SPM, and the proposed PTS scheme uses a hybrid SPM, where the input data block \( X \) is partitioned into \( M = 4 \) subblocks. The elements in the vector of rotation factors \( b_j = [1, b_M^1, b_M^2, \ldots, b_M^M]^T \) are selected from the set \( \{ \pm 1, \pm j \} \). The number of candidate signals is \( C = 64 \) for the CPTS and the RC-PTS scheme.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Rotation Factor</th>
<th>Complex ( \alpha )</th>
<th>Real ( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPTS</td>
<td>( (M-1) LN )</td>
<td>0.5M ( N ) ( \log_2 M ) + CLN</td>
<td>2M ( N ) ( \log_2 M ) + CLN</td>
</tr>
<tr>
<td>RC-PTS</td>
<td>( (M-1)pn LN )</td>
<td>0.5M ( N ) ( \log_2 M ) + MN ( N ) ( \log_2 N ) + CNL</td>
<td>2M ( N ) ( \log_2 M ) + MN ( N ) ( \log_2 N ) + CNL</td>
</tr>
<tr>
<td>Proposed</td>
<td>( 0.5 { (M-1)p_n LN } )</td>
<td>0.5M ( N ) ( \log_2 M ) + CLN</td>
<td>2M ( N ) ( \log_2 M ) + CLN</td>
</tr>
</tbody>
</table>

### 6.1 Comparison of PAPR Reduction Performance

Figure 2 shows the complementary cumulative distribution function (CCDF) of the CCDF of the PTS scheme and the proposed PTS scheme, where the former uses the pseudo-random, the adjacent, and the interleaving SPM. The proposed PTS scheme does not use the cost functions, and the number of candidate signals is increased from 32 to 64 when \( x_1^{(1)} \) derived from (18) is used to generate more candidate signals. Figure 2 shows that, when CCDF = 10\(^{-3}\), the proposed PTS scheme with \( C = 64 \) is about 4 dB, which is only a 0.15 dB degradation of that of the CPTS scheme with the pseudo-random SPM. Also, the proposed PTS scheme with \( C = 32 \) has a better PAPR reduction performance than the CPTS schemes with the adjacent and the interleaving SPM.

Figure 3 shows the PAPR reduction performance of the proposed PTS schemes with cost functions and various thresholds \( \alpha = \frac{\Phi_N}{\Phi_{N-64}} \) and \( \phi_{N-64} = 2.67 \). The CPTS scheme with \( C = 32 \) and 64 are shown for comparison. Figure 3 shows that the proposed PTS scheme with \( \lambda = 0.44 \) has almost the same PAPR reduction performance as the case that uses all samples to estimate the peak power, in which \( \alpha \leq 1.18 \) and the corresponding \( p_N \geq 0.311 \).

### 6.2 Comparison of Computation Complexity

The proposed PTS scheme and the RC-PTS scheme use some of the samples to estimate the peak power of each can-
Fig. 2 Comparison of PAPR reduction performance for the CPTS scheme with three SPMs and the proposed PTS scheme with \( C = 32 \) and 64 candidate signals.

Fig. 3 Comparison of PAPR reduction performance for the CPTS, the RC-PTS of [17], and the proposed PTS scheme with/without cost functions, where \( C = 64 \) for the latter two schemes and the thresholds \( \alpha = 0.67, 1.18, \) and 1.34 for the proposed scheme.

didate signal. We choose the threshold \( \alpha = 1.18 \) with corresponding \( p_{\alpha} = 0.311 \) for these two schemes. Table 2 illustrates the comparison of computation complexity of various PTS schemes derived from Table 1. The computation complexity of the CPTS scheme with \( C = 64 \) is taken as 100%. We note that when \( N = 64, L = 4, M = 4, \) and \( C = 64 \), the proposed PTS scheme needs 7,360 complex multiplications and 23,487 real additions, which are only about 36% and 18% of the CPTS scheme with \( C = 64 \), respectively. The RC-PTS scheme needs about 50% of the complex multiplications and 40% of the real additions of the CPTS scheme. It is clear that the proposed PTS scheme has much lower computation complexity than the other two works using the pseudo-random SPM. In addition, the CPTS scheme with \( C = 13 \) has a close percentage of complex multiplications as the case of proposed PTS scheme with \( \alpha = 1.18 \), but with worse PAPR reduction performance shown in Fig. 3.

Table 2 Comparison of computation complexity for various PTS schemes with \( N = 64, L = 4, M = 4, \) and \( C = 64 \).

<table>
<thead>
<tr>
<th>Schemes</th>
<th>( \alpha )</th>
<th>( p_{\alpha} )</th>
<th>Complex ( \times^* )</th>
<th>Real ( \times^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPTS (( C=64 ))</td>
<td>NA</td>
<td>NA</td>
<td>20,480 (100%)</td>
<td>13,1071 (100%)</td>
</tr>
<tr>
<td>CPTS (( C=13 ))</td>
<td>NA</td>
<td>NA</td>
<td>7424 (36.3%)</td>
<td>3967 (30.3%)</td>
</tr>
<tr>
<td>RC-PTS in [17]</td>
<td>1.18</td>
<td>0.311</td>
<td>10,176 (50%)</td>
<td>52,799 (40%)</td>
</tr>
<tr>
<td>Proposed PTS</td>
<td>0.69</td>
<td>0.720</td>
<td>14,080 (69%)</td>
<td>43,647 (33%)</td>
</tr>
<tr>
<td></td>
<td>1.18</td>
<td>0.311</td>
<td>7,360 (36.0%)</td>
<td>23,487 (18.0%)</td>
</tr>
<tr>
<td></td>
<td>1.34</td>
<td>0.220</td>
<td>5,888 (29%)</td>
<td>19,071 (15%)</td>
</tr>
</tbody>
</table>

7. Conclusions

In this paper, we have proposed a new low-complexity PTS scheme that uses a hybrid SPM to reduce the PAPR of OFDM systems. The hybrid SPM is a combination of the pseudo-random and interleaving methods; it reduces the complexity of the IFFTs and the optimization process used to generate candidate signals. We further enhance the PAPR reduction performance of the proposed scheme by multiplying a selected subblock by a predefined phase rotation vector to form a new frequency-domain subblock, and then use the new subblock to generate more candidate signals for selection. The IFFT output of the new subblock can be obtained easily by performing a circularly-shifting-left operation on the IFFT output sequence of the selected subblock. Simulation results show that the proposed PTS scheme achieves a PAPR reduction performance close to that of the CPTS scheme and the RC-PTS scheme with the pseudo-random SPM, but has much lower computation complexity.

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References

KU et al.: A LOW-COMPLEXITY PTS SCHEME WITH THE HYBRID SUBBLOCK PARTITION METHOD FOR PAPR REDUCTION IN OFDM SYSTEMS


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