Low-Complexity PTS-Based Schemes for PAPR Reduction in SFBC MIMO-OFDM Systems

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Abstract—Multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) is an attractive technique for high-data-rate transmission. However, MIMO-OFDM systems have an inherent drawback in that the transmitted signals may suffer from a high peak-to-average power ratio (PAPR). The conventional partial transmit sequences (PTS) scheme can be applied to each transmitting antenna directly to reduce the PAPR of MIMO-OFDM systems, but it has high computational complexity. In this paper, two types of low-complexity PTS schemes are proposed to reduce the PAPR for MIMO-OFDM systems that use space-frequency block coding (SFBC). The two proposed PTS schemes use the sample powers of subblocks to generate cost functions for selecting samples to estimate the peak power of each candidate signal, thus reducing the computational complexity. In addition, for one of the proposed PTS schemes, the similarity of the input signals from the various transmitting antennas is used to further reduce the computational complexity. Simulation results show that, as compared with other related works, the proposed PTS schemes can achieve a reduced PAPR performance and a bit error rate performance close to that of SFBC MIMO-OFDM systems, but at much lower computational cost.

Index Terms—Multiple-input multiple-output (MIMO), orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), partial transmit sequences (PTS).

I. INTRODUCTION

MULTIPLE-INPUT multiple-output (MIMO) is a promising technology for improving the performance of wireless communication. Both the throughput of data transmission and the efficiency of spectrum utilization can be simultaneously increased through the use of multiple antennas at both the transmitter and receiver. Orthogonal frequency division multiplexing (OFDM) is an effective method for encoding digital data on multiple carrier frequencies, providing robustness against the multipath fading channels. The combination of MIMO and OFDM techniques, namely, MIMO-OFDM, has become one of the most attractive candidates for broadband wireless communications [1], such as wireless metropolitan area networks (WMANs) [2] and 3GPP Long Term Evolution (LTE) systems [3]. However, similar to the problem encountered in OFDM systems, the signals transmitted in MIMO-OFDM systems may suffer from undesired distortion due to a high peak-to-average power ratio (PAPR) of the system when the signals pass through a nonlinear device such as a high-power amplifier (HPA).

Numerous PAPR reduction methods have been proposed for OFDM systems [4], including precoding [5], companding transform [6], [7], tone reservation [8], [9], active constellation extension (ACE) [10], selected mapping (SLM) [11], [12], and partial transmit sequences (PTS) [13]–[15]. Also, there are many research reports on the reduction of PAPR for MIMO-OFDM systems in the literature [16]–[20]. Among these methods, the PTS-based technique has drawn wide attention due to its feasibility in reducing the PAPR of OFDM and MIMO-OFDM systems. In the conventional PTS scheme, the input data block is partitioned evenly into disjointed subblocks. The inverse fast Fourier transform (IFFT) outputs of the subblocks are weighted by a set of rotation factors and then added to form various candidate signals, where the signal with the lowest PAPR value is transmitted. The conventional PTS scheme is a non-distortion technique and has a good PAPR reduction performance, but it also has high computational complexity and needs to send side information to the receiver. Many methods have been proposed to resolve these two main drawbacks of the PTS technique [13]–[15]. For MIMO-OFDM systems, the cooperative PTS (co-PTS) method presented in [20] uses spatial subblock circular permutations across all the transmitting antennas for the odd subblocks to increase the number of candidate signals, and then uses alternate optimization to find the signal with the lowest PAPR value. Although the co-PTS method has a good PAPR reduction performance for MIMO-OFDM systems, it must use all the samples of each candidate signal for peak power estimation.

The ACE method also is attractive for reducing the peak power of OFDM signals by extending some modulation constellation points toward the outside of the constellation without loss of data rate. In contrast to the PTS technique, the ACE method usually has a better PAPR reduction performance and does not need to send side information to the receiver. However, the ACE method uses a complex repeated-clipping-and-filtering process and has a power penalty. Furthermore, when the ACE method is applied to the input signal of an individual transmitting antenna in MIMO-OFDM systems with...
space-frequency block coding (SFBC), it may destroy the orthogonality among the input signals of various antennas.

In this paper, we propose two types of low-complexity PAPR reduction methods (referred to as type I and type II) based on the PTS technique for SFBC MIMO-OFDM systems. These two proposed PTS schemes use different subblock partition methods to partition the input data blocks evenly into several subblocks. The cost function is evaluated by adding the power of the samples with the same time index in the subblocks. Only the samples with a cost function value greater than a predefined threshold are used to estimate the PAPR of the candidate signals in each antenna. The computational complexity is thus reduced by using the selected samples for peak power estimation. Also, the similarity of the time-domain signals from different antennas is used to develop algorithms to further reduce the computational complexity of evaluating the cost function and candidate signals. Simulation results show that the two proposed PTS schemes can achieve a PAPR reduction performance and bit error rate (BER) performance close to that of MIMO-OFDM systems using the conventional PTS or the co-PTS scheme, but with much lower computational complexity.

The remainder of this paper is organized as follows. In Section II, the basic concept of PAPR in SFBC MIMO-OFDM systems is introduced. The basic ideas behind the two proposed low-complexity PTS-based schemes for SFBC MIMO-OFDM systems are described in Section III. The computational complexity of the proposed PTS schemes as well as two related works is analyzed in Section IV. In Section V, simulation results are presented to compare the PAPR reduction performance, the BER performance, and the computational complexity of the proposed PTS schemes with related works. Finally, a brief conclusion is drawn in Section VI.

II. BACKGROUND

For simplicity, our discussion in this section focuses on MIMO-OFDM systems with two transmitting antennas. In SFBC MIMO-OFDM systems with two transmitting antennas, the input data block \( \mathbf{X} = [X_0, X_1, \ldots, X_{N-1}]^T \) is encoded with the Alamouti space-frequency encoder into two vectors, as follows:

\[
\begin{align*}
\mathbf{X}_1 &= [X_0, -X_1^*, \ldots, -X_{N-2}^*, X_{N-1}^*]^T \\
\mathbf{X}_2 &= [X_1, X_0^*, \ldots, X_{N-1}^*, X_{N-2}^*]^T
\end{align*}
\]

(1)

where \( N \) is the number of subcarriers and \((\cdot)^*\) denotes the complex conjugate operation [21]. The continuous-time baseband OFDM signal of \( \mathbf{X}_i \) for \( i = 1, 2 \) can be represented as

\[
x_i(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_{i,k} e^{j2\pi k \Delta f t}, \quad 0 \leq t \leq T
\]

(2)

where \( X_{i,k} \) represents the data symbol modulated by the \( k \)th subcarrier for \( k = 0, 1, \ldots, N-1 \) in transmitting antenna \( \text{TX}_i \) with \( i = 1, 2 \). \( \Delta f \) is the frequency difference between subcarriers, and \( T \) is the OFDM symbol duration. The PAPR of \( x_i(t) \) in (2) is defined as the ratio of the maximum power to the average power of \( x_i(t) \) and is expressed as

\[
PAPR_i = \max_{0 \leq t \leq T} \frac{|x_i(t)|^2}{E[|x_i(t)|^2]} \quad (3)
\]

where \( E[\cdot] \) denotes the expected value operation. The discrete-time OFDM signal \( x_i = [x_{i,0}, x_{i,1}, \ldots, x_{i,N-1}]^T \) in antenna \( \text{TX}_i \) is usually generated by sampling \( x_i(t) \) with the rate \( N/T \) and can be expressed as

\[
x_{i,n} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_{i,k} e^{j2\pi kn/\pi} \quad n = 0, 1, \ldots, N-1
\]

(4)

which can be realized by using an \( N \)-point IFFT. However, \( x_{i,n} \) obtained by (4) may miss the peak power of \( x_i(t) \) and result in an overly optimistic PAPR estimation. It was verified in [9] that the PAPR estimated from the samples using \( L = 4 \) times oversampling can closely approximate that of (3). Because the two signals \( x_1 \) and \( x_2 \) are correlated and transmitted simultaneously from antennas \( \text{TX}_1 \) and \( \text{TX}_2 \), respectively, the PAPR of SFBC MIMO-OFDM systems should be considered jointly as

\[
PAPR(x_1, x_2) = \max\{PAPR(x_1), PAPR(x_2)\} \quad (5)
\]

One of the main drawbacks of MIMO-OFDM systems is that the transmitted signals may have a high PAPR. Many PAPR reduction techniques have been presented for MIMO-OFDM systems [16]-[20]. The conventional PTS scheme, as shown in Fig. 1, applied directly to each of the transmitting antennas of the MIMO-OFDM system, is referred to as the ordinary PTS (oPTS) scheme. The oPTS scheme has a good PAPR reduction performance, but it suffers from high computational complexity. In the cooperative PTS (co-PTS) scheme presented in [20], the authors use alternate optimization to reduce the number of multiplications of rotation factors and use spatial subblock circular permutations between antennas to increase the number of candidate signals. Both the oPTS and co-PTS methods show a good PAPR reduction performance for MIMO-OFDM systems; however, they use all the samples to estimate the peak power of candidate signals in the optimization process.

III. PROPOSED PTS-BASED SCHEMES FOR SFBC MIMO-OFDM SYSTEMS

In this section, we propose two types of PTS-based schemes to resolve the high-PAPR problem for SFBC MIMO-OFDM systems.
systems. We first discuss the application of the proposed schemes to systems with $N_T = 2$ transmitting antennas, and then the application of the proposed schemes to systems with $N_T = 4$ transmitting antennas.

A. Proposed Type I PTS Scheme

In the proposed type I PTS scheme, the conventional PTS scheme is applied to each transmitting antenna in SFBC MIMO-OFDM systems. The data block $X_i$ of (1) is first partitioned evenly into $M_b$ disjoint subblocks, denoted as

$$X_{i,m} = [X_{i,m,0}, X_{i,m,1}, \ldots, X_{i,m,N-1}]^T$$

(6)

where $i = 1, 2$ and $1 \leq m \leq M_b$. The $e$th candidate signal $x^e_i = [x^e_{i,0}, x^e_{i,1}, \ldots, x^e_{i,N-1}]^T$ formed by using the subblocks of (6) is expressed as

$$x^e_i = \sum_{m=1}^{M_b} \text{IFFT}\{b^e_{i,m}X_{i,m}\} = \sum_{m=1}^{M_b} b^e_{i,m}\text{IFFT}\{X_{i,m}\}$$

(7)

where $x_{i,m} = \text{IFFT}\{X_{i,m}\} = [x_{i,m,0}, x_{i,m,1}, \ldots, x_{i,m,N-1}]^T$ represents the IFFT of subblock $X_{i,m}$ for $i = 1, 2$, and $1 \leq m \leq M_b$; $b^e_{i,m}$ is the rotation factor selected from a $W$-element set $\Theta = \{e^{j\theta}|i = 0, 1, \ldots, W - 1\}$ for $1 \leq c \leq C$; and $C(\leq W^{M_b})$ is the number of candidate signals of each antenna. The optimal output signal $x^e_{i,opt}$ for $i = 1, 2$ is selected from the $C$ candidate signals by using the minimax criterion as follows:

$$\left[ x^e_{1,opt}, x^e_{2,opt} \right] = \min_{1 \leq e \leq C} \{ \max\{\text{PAPR}(x^e_{1}), \text{PAPR}(x^e_{2})\} \}.$$  

(8)

We observe that only the peak power of each candidate signal is necessary to find the optimal signals in (8). In [13], an algorithm was presented to select samples for estimating the peak power of the candidate signals in the conventional PTS scheme. Based on the concept in [13], a cost function is defined for antenna $T_X$ as

$$Q_{i,n} = \sum_{m=1}^{M_b} |x_{i,m,n}|^2, n = 0, 1, \ldots, N - 1$$

(9)

where $x_{i,m,n}$ is the sample in $x_{i,m}$ located at time $n$ for $i = 1, 2$, respectively. It is derived in [13] that only the samples with $Q_{i,n}$ greater than a predefined threshold $\alpha_{TH}$ are used to estimate the peak power of candidate signals for antenna $T_X$ with $i = 1, 2$. Therefore, the computational complexity of the optimization process is reduced. However, the correlation between $X_1$ and $X_2$ of (1) is not used in (9). In the following, another type of PTS scheme is developed for the SFBC MIMO-OFDM system, where a sample-selection algorithm is derived for selecting the samples jointly to estimate the peak power of the candidate signals of the two antennas.

B. Proposed Type II PTS Scheme

In the proposed type II PTS scheme, the two input signals $X_1$ and $X_2$ of (1) are decomposed into even and odd parts, respectively, expressed as

$$X_1^0 = [X_0, 0, X_2, \ldots, X_{N-2}]^T$$

(10.a)

$$X_1^1 = [0, -X_1^*, 0, \ldots, -X_{N-1}^*]^T$$

(10.b)

$$X_2^0 = [X_1, 0, X_3, \ldots, X_{N-1}]^T$$

(10.c)

$$X_2^1 = [0, X_0^*, 0, \ldots, X_{N-2}^*]^T$$

(10.d)

where $X_1^0$ and $X_1^1$ represent the even and odd parts of $X_i$, respectively, for $i = 1, 2$. It is clear $X_1^2$ that can be obtained by performing shift-right and conjugate operations on $X_1^0$, and that $X_2^0$ can be obtained by performing shift-left, negate, and conjugate operations on $X_1^1$. In addition, each of the four signals of (10) is partitioned evenly into $M$ subblocks by using the adjacent subblock partition method. Each subblock is denoted as $X_{i,m}^e$ with $e = 0, 1, i = 1, 2$, and $1 \leq m \leq M$. For example, in the case of $M = 2$, the subblocks obtained from $X_1^0$ and $X_1^1$ for the antenna $T_{X_1}$ are represented as

$$X_{1,1,1}^0 = [X_0, 0, \ldots, X_{N/2-2}, 0, \text{zeros}(N/2)]^T$$

(11.a)

$$X_{1,1,2}^0 = [\text{zeros}(N/2), X_{N/2}, 0, \ldots, X_{N-2}, 0]^T$$

(11.b)

$$X_{1,1,1}^1 = [0, -X_1^*, 0, \ldots, -X_{N/2-1}^*, \text{zeros}(N/2)]^T$$

(11.c)

$$X_{1,1,2}^1 = [\text{zeros}(N/2), 0, -X_{N/2+1}^*, \ldots, -X_{N-1}^*]^T$$

(11.d)

where $\text{zeros}(N/2)$ denotes a vector that contains $N/2$ zeros. Similarly, the subblocks obtained from $X_2^0$ and $X_2^1$ for antenna $T_{X_2}$ are represented as

$$X_{2,1,1}^0 = [X_1, 0, \ldots, X_{N/2-1}, 0, \text{zeros}(N/2)]^T$$

(12.a)

$$X_{2,1,2}^0 = [\text{zeros}(N/2), X_{N/2+1}, 0, \ldots, X_{N-1}, 0]^T$$

(12.b)

$$X_{2,1,1}^1 = [0, X_0^*, \ldots, 0, X_{N/2-2}^*, \text{zeros}(N/2)]^T$$

(12.c)

$$X_{2,1,2}^1 = [\text{zeros}(N/2), 0, X_{N/2+2}^*, \ldots, 0, X_{N-2}^*]^T$$

(12.d)

Then, for input data block $X_i$ with $i = 1, 2$, the $e$th candidate signal formed by using the subblocks generated by (10) is expressed as

$$x^e_i = [x^e_{i,0}, x^e_{i,1}, \ldots, x^e_{i,N-1}]^T$$

$$\sum_{m=1}^{M} \text{IFFT}\{b^e_{i,m}x^0_{i,m} + b^e_{i,m}x^1_{i,m}\}$$

(13)

where $x^e_{i,m} = \text{IFFT}\{X^e_{i,m}\} = [x^e_{i,m,0}, x^e_{i,m,1}, \ldots, x^e_{i,m,N-1}]^T$ represents the IFFT of subblock $X^e_{i,m}$ for $i = 1, 2, e = 0, 1$, and $1 \leq m \leq M$; $b^e_{i,m}$ is the rotation factor of subblock $X^e_{i,m}$ for $1 \leq c \leq C$; and $C(\leq W^{M_b})$ is the number of candidate signals of each antenna. The optimal output signals $x^e_{i,opt}$ for $i = 1, 2$ are determined by using (8).
In the proposed type II PTS scheme, the cost function $Q_{i,n}$ of the sample located at time $n$ in the transmitting antenna $T_X_i$ is defined by

$$Q_{i,n} = \sum_{m=1}^{M} \left( |x_{i,m,n}^0|^2 + |x_{i,m,n}^1|^2 \right), \quad n = 0, 1, \ldots, N-1$$

(14)

where $x_{i,m,n}^0$ and $x_{i,m,n}^1$ are the samples of $x_{i,m}^0$ and $x_{i,m}^1$.

Furthermore, by using Property 2 of the time-domain signal presented in [18], $x_{i,m}^0$ and $x_{i,m}^1$ have the form of

$$x_{i,m}^0 = \left[ x_{i,m,0}^0 \ x_{i,m,1}^0 \cdots x_{i,m,M}^0 \right]^T$$

(15.a)

$$x_{i,m}^1 = \left[ x_{i,m,0}^1 - \hat{x}_{i,m}^1 \ x_{i,m,1}^1 - \hat{x}_{i,m}^1 \cdots x_{i,m,M}^1 - \hat{x}_{i,m}^1 \right]^T$$

(15.b)

for $i = 1, 2$ and $1 \leq m \leq M$, where $\hat{x}_{i,m}$ denotes the vector that contains the elements of the first half of $x_{i,m}^e$ for $e = 0, 1$. Therefore, it is easy to determine that the vector of cost function $\hat{Q}_i = [\hat{Q}_{i,0}, \hat{Q}_{i,1}, \ldots, \hat{Q}_{i,N-1}]^T$ has the form

$$\hat{Q}_i = \left[ \hat{Q}_i \ \hat{Q}_i \right]^T$$

(16)

where $\hat{Q}_i$ denotes the first half of the elements of $Q_i$ for $i = 1, 2$. Thus, it is necessary to compute only $Q_{i,n}$ with $n = 0, 1, \ldots, N/2 - 1$ by using (14). Also, the subblocks of $X_{2,n}^e$ can be obtained from the subblocks of $X_{1,n}^e$ using conjugate/negative/cyclic shift operations. For example, $X_{2,1}^1$ of (12.c) can be obtained by performing cyclic shift-right and conjugate operations on $X_{1,1}^0$ of (11.a). Because $Q_{i,n}$ in (14) is generated by using the time-domain signals $x_{i,m}^e$, the relationship between $Q_i$ and $Q_{i,n}$ can be obtained easily by using the linearity and shifting properties of IFFT, as follows:

$$Q_2 = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & \ldots & 0 \end{bmatrix} \times \hat{Q}_1 = JQ_1$$

(17)

From (16) and (17), only the $N/2$ elements of $\hat{Q}_1$ are computed by using (14) for the proposed type II PTS scheme; therefore, the overhead of generating the cost function is reduced. Fig. 2 shows examples of $Q_1$, $Q_2$, and $JQ_1$ for the SFBC MIMO-OFDM systems with $N = 64$ subcarriers, in which $Q_2$ is the same as $JQ_1$, and the first half is the same as the second half in each figure.

The cost functions in $Q_1$ and $Q_2$ are used to select samples for estimating the peak power of candidate signals in antennas $T_X_1$ and $T_X_2$, respectively. A flag vector $f_i = [f_{i,0}, f_{i,1}, \ldots, f_{i,N-1}]^T$ for $i = 1, 2$ is used to store the results of the comparison of the cost function $Q_{i,n}$ and a predefined threshold $\alpha_{TH}$, where the elements of $f_i$ are defined by

$$f_{i,n} = \begin{cases} 1, & \text{if } Q_{i,n} \geq \alpha_{TH} \\ 0, & \text{if } Q_{i,n} < \alpha_{TH} \end{cases} \quad \text{for } n = 0, 1, \ldots, N-1.$$ 

(18)

Only the samples $x_{i,n}^e$ with $f_{i,n} = 1$ are used to estimate the peak power of candidate signal $x_i^e$. Because $f_i$ is generated by using $Q_i$, the first and second halves of the elements of $f_i$ are the same and $f_2 = Jf_1$, similar to the formats shown in (16) and (17). According to the above discussion, the steps of the proposed type II PTS scheme for the SFBC MIMO-OFDM systems are summarized as follows:

1. Decompose the two input data block signals $X_1$ and $X_2$ of (1) into even and odd parts, respectively, as shown in (10).
2. Partition each of the four data blocks obtained in step 1) evenly into $M$ disjoint subblocks, denoted as $X_{i,m}^e$ with $e = 0, 1, i = 1, 2$, and $1 \leq m \leq M$.
3. Find $X_{i,m}^e = IFFT[X_{i,m}^e] = [x_{i,m,0}^e \ x_{i,m,1}^e \cdots x_{i,m,N-1}^e]^T$ for $e = 0, 1, i = 1, 2$, and $1 \leq m \leq M$.
4. Compute $Q_{i,n}$ of $Q_i$ for $n = 0, 1, \ldots, N/2 - 1$ by using (14).
5. Use $Q_{i,n}$ of step 4) to generate the flag $f_{i,n}$ in (18) for $n = 0, 1, \ldots, N/2 - 1$, as well as $f_1$ and $f_2$.
6. For the candidate signal $x_i^e$ of antenna $T_X_i$, only the samples $x_{i,n}^e$ with $f_{i,n} = 1$ are generated for estimating the PAPR of $x_i^e$, where $1 \leq e \leq C, i = 1, 2$, and $n = 0, 1, \ldots, N/2 - 1$.
7. Determine the optimal output signals $x_i^{opt}$ and $x_2^{opt}$ for transmission.
8. Generate all samples of the optimal output signals $x_i^{opt}$ and $x_2^{opt}$ for transmission.

Fig. 3 shows part of the block diagram of the proposed type II PTS scheme that processes $X_1$ for antenna $T_X_1$ in the SFBC MIMO-OFDM systems with two transmitting antennas. $f_2$ is used to select the samples to estimate the peak power of the candidate signals for antenna $T_X_2$. The part used to process $X_2$ is similar to that for $X_1$, and so is not shown here. The block of phase optimization represents the process of (8) to determine the optimal output signals for both antennas jointly.

The major difference between the proposed type I and type II PTS schemes is that the input data blocks $X_1$ and $X_2$ in the type I PTS scheme are not divided into even and odd parts, so there is no similarity between or in the cost functions and the time-domain signals of the subblocks of the type I PTS scheme. The block diagram of the proposed type I PTS scheme is similar to that shown in Fig. 3, except for the generation of $f_2$. 

![Fig. 2. Example of $Q_1$, $Q_2$, and $JQ_1$ for the SFBC MIMO-OFDM systems with $N = 64$ subcarriers.](image)
C. MIMO-OFDM Systems With Four Transmitting Antennas

The two proposed PTS schemes applied to SFBC MIMO-OFDM systems with $N_T = 4$ transmitting antennas is discussed in this subsection. The input data block $X$ is encoded into a code matrix $D$ by using the encoder proposed in [22], expressed as

$$D = [X_1 \ X_2 \ X_3 \ X_4] = \left[ D_1^T \ D_2^T \ldots \ D_{N/4}^T \right]^T$$

(19)

where $D_v$, $1 \leq v \leq N/4$, denotes the $v$th 4-by-4 code matrix in the form of

$$D_v = \begin{bmatrix}
X_{3(v-1)} & X_{3(v-1)+1} & X_{3(v-1)+2} & X_{3(v-1)+3} \\
-\frac{1}{2}X_{4(v-1)+1} & \frac{1}{2}X_{4(v-1)} & -\frac{1}{2}X_{4(v-1)+3} & \frac{1}{2}X_{4(v-1)+2} \\
-\frac{1}{2}X_{4(v-1)+2} & \frac{1}{2}X_{4(v-1)+3} & X_{4(v-1)} & -\frac{1}{2}X_{4(v-1)+1} \\
X_{4(v-1)+3} & -\frac{1}{2}X_{4(v-1)+2} & -\frac{1}{2}X_{4(v-1)+1} & X_{4(v-1)}
\end{bmatrix} T$$

(20)

In the SFBC MIMO-OFDM systems with $N_T = 4$, the proposed type I PTS scheme uses the conventional PTS scheme in each transmitting antenna. In the proposed type II PTS scheme, the input data block $X_i$, $1 \leq i \leq 4$, of transmitting antenna $T_{X_i}$ is first partitioned into four subblocks by using the interleave subblock partition method (SPM). Each subblock is further partitioned into two subblocks by using the adjacent SPM. Each of the eight subblocks derived from $X_i$, denoted as $X_{i,m}$ with $1 \leq m \leq 8$, have $N/8$ nonzero elements in the first or the second half of the subblock.

Similarly, a set of cost functions $Q_{i}^4 = [Q_{i,0}^4 \ Q_{i,1}^4 \ldots \ Q_{i,N-1}^4]^T$ for transmitting antenna $T_{X_i}$ with $1 \leq i \leq 4$ is generated by using $X_{i,m}$, the IFFT of $X_{i,m}$, expressed by

$$Q_{i,n}^4 = \sum_{m=1}^{8} |X_{i,m,n}|^2, n = 0, 1, \ldots, N - 1.$$  

(21)

By the properties of IFFFT, it is easy to determine that, in the proposed type II scheme, $Q_{i}^4$ has the form of

$$Q_{i}^4 = [Q_{i,0}^4 \ Q_{i,1}^4 \ldots \ Q_{i,3}^4]^T$$

(22)

where $Q_{i,0}^4$ denotes the first quarter of the elements of $Q_{i}^4$ for $1 \leq i \leq 4$. Also, $Q_{i,1}^4 = Q_{i,2}^4 = Q_{i,3}^4$, and $Q_{i,4}^4 = JQ_{i,3}^4$.

Thus, only the elements of the first quarter of $Q_{i}^4$ are generated and used to select the samples to estimate the peak power of the candidate signals in each transmitting antenna. The usage of cost functions in the two proposed schemes for SFBC MIMO-OFDM systems with $N_T = 4$ is similar to that presented in the previous subsection, so it is not discussed here.

IV. ANALYSIS OF COMPUTATIONAL COMPLEXITY

A. Selection of the Threshold of Cost Function

In the two proposed PTS schemes for SFBC MIMO-OFDM systems, only the samples with the corresponding cost function greater than or equal to the threshold $\alpha_{TH}$ are used to estimate the peak power of the candidate signals in each transmitting antenna. Thus, the choice of $\alpha_{TH}$ determines the computational complexity of the proposed methods. In [13], the authors derived the minimum possible peak power $\Phi_N$ in the OFDM system with $N$ subcarriers as follows:

$$\Phi_N = -\sigma^2 \ln \left[ -N \left( \frac{\alpha}{\Phi N} \right)^{0.5} \right]$$

(23)

where $\sigma^2$ is the average power of OFDM signals and $\gamma$ is the predefined lower bound of the probability that the peak power of the OFDM symbols is greater than or equal to $\Phi_N$. When the threshold $\alpha_{TH} = \Phi_N/M_b$ is used to select samples for peak power estimation in the PTS scheme with $M_b$ subblocks, the probability of missing the exact peak power is less than $1 - \gamma$.

Let $p_{\sigma}$ be the probability that the elements of $Q_i$ are greater than the threshold $\alpha_{TH}$; then there are $p_{\sigma}N$ samples used in the proposed PTS schemes to estimate the peak power of each candidate signal in each antenna. The derivation of $p_{\sigma}$ can be found in [13]. In this paper, we investigate the PAPR reduction performance of the proposed PTS schemes with various thresholds $\alpha_{TH} = \lambda \Phi_N$, where $\gamma = 0.9999$ and $\lambda \geq 1/M_b$.

B. Computational Complexity of Optimization Process

The goal of the two proposed schemes is to reduce the computational complexity of the optimization process of the PTS scheme used in SFBC MIMO-OFDM systems. Thus, the computational complexity of the optimization process is analyzed for various PTS techniques. Assume that the rotation factor of the first subblock is 1. If the oPTS scheme has $M_b$ subblocks and $C$ candidate signals in each antenna, it needs $C(M_b - 1)/N$ multiplications of rotation factors, $C(M_b - 1)/N$ complex additions in (7), $CN$ complex multiplications to compute the sample powers, and $CN - 1$ comparisons to find the optimal candidate signal of each antenna. Finally, the min-max criterion of (8) need $2C - 1$ comparisons. In the co-PTS method [20], only the subblocks with even numbers are multiplied by the rotation factors in (7); the subblocks with odd numbers are rotated among various antennas. Thus, the number of multiplications of rotation factors is reduced to $\lambda^{0.5} M_b$ if $M_b$ is an even number. The other terms used to determine the computational complexity of the co-PTS scheme are the same as those of the oPTS scheme.
In the proposed type I PTS scheme, generation of the cost function in (9) needs \( M_bN \) complex multiplications and \((M_b - 1)N\) additions. Also, \( N \) comparisons with the threshold value in (18) are used to select the samples for peak power estimation, in which \( p_aN \) samples are selected in each candidate signal. During the optimization process, each antenna needs \( C(M_b - 1)p_aN \) multiplications of rotation factors and \( C(M_b - 1) \) complex additions to form \( C \) candidate signals, \( Cp_aN \) complex multiplications to compute the sample powers, and \( Cp_aN - 1 \) comparisons to find the optimal signal of each antenna. Finally, the minimax criterion of (8) needs \( 2C - 1 \) comparisons.

In the proposed type II PTS scheme, the even and odd parts of \( \mathbf{X}_1 \) and \( \mathbf{X}_2 \) in (10) are partitioned into \( M(= M_b/2) \) sub-blocks, respectively. The \( N/2 \) elements of \( \mathbf{Q}_1 \) are generated by using (14), which needs \( M_bN/2 \) complex multiplications and \((M_b - 1)N/2\) real additions. Only \( N/2 \) comparisons are used to generate the flag vectors \( \mathbf{f}_1 \) and \( \mathbf{f}_2 \) in (18). Therefore, the computational complexity of using cost functions to select samples in the proposed type II PTS scheme is only one-fourth that of the proposed type I PTS scheme when two transmitting antennas are considered jointly, where \( p_aN \) samples are selected to estimate the peak power of candidate signals in each antenna.

In addition, the similarity between the first and the second part of \( \mathbf{x}_{\ell,m} \) shown in (15) can be applied to (13) to further reduce the computational complexity of generating \( \mathbf{x}_\ell^c \), as below

\[
\mathbf{x}_\ell^c = \sum_{m=1}^{M} \left( b_{\ell,m}^{c,0} \mathbf{x}_{\ell,m} + b_{\ell,m}^{c,1} \mathbf{x}_{\ell,m} \right)
= \sum_{m=1}^{M} \left( b_{\ell,m}^{c,0} \mathbf{x}_{\ell,m}^0 + b_{\ell,m}^{c,1} \mathbf{x}_{\ell,m}^1 \right) \mathbf{x}_{\ell,m}^T
= \sum_{m=1}^{M} \left[ b_{\ell,m}^{c,0} \mathbf{x}_{\ell,m}^0 + b_{\ell,m}^{c,1} \mathbf{x}_{\ell,m}^1 \right].
\]

Because the two summation terms in the first half can be reused in the second half of (20), the generation of \( C \) candidate signals needs \( C(M_b - 1)p_aN/2 \) multiplications of rotation factors and \( CM_b p_aN/2 \) complex additions by (20). Table I provides a comparison of the computational complexity of the optimization process of one antenna for various PTS schemes, where the term \( N \) is replaced by \( LN \) when \( L \) times oversampling is considered. If the computational complexity of two antennas is considered jointly, each term in Table I should be multiplied by a factor of two, except for the terms marked with an asterisk. Note that a complex addition is equivalent to two real additions and a comparison is equivalent to a real addition.

Also, in the type II PTS scheme, the time-domain signals \( x_{\ell,m}^c \) of antenna \( T \mathbf{X}_2 \) can be obtained from \( x_{\ell,m}^c \) of \( T \mathbf{X}_1 \) by using simple conjugate/cyclic shift operations for \( e = 0, 1, \) and \( 1 \leq m \leq M \). Also, \( x_{\ell,m}^c \) of (15) can be evaluated by using \( N/2 \)-point IFFT operations. Thus, the proposed type II PTS scheme has much fewer IFFT computations than the other three works. The discussion about the properties of time-domain signals can be found in [18].

C. Removal of Redundant Candidate Signals for the Proposed Type II PTS Scheme

Various rotation factors \( b_{\ell,m}^{c,0} \) are applied to \( x_{\ell,m}^c \) to generate the candidate signal \( \mathbf{x}_\ell^c \) in (13). \( b_{\ell,m}^{c,0} \) is usually selected from the set \( \Theta = \{ \pm 1, \pm j \} \) to simplify the multiplication operation of rotation factors and \( b_{\ell,m}^{c,0} \) is fixed, without degrading the PAPR reduction performance. However, two candidate signals \( \mathbf{x}_\ell^c \) and \( \mathbf{x}_\ell^e \) have the same PAPR value if the corresponding rotation factors \( b_{\ell,m}^{c,0} = b_{\ell,m}^{e,0} \) and \( b_{\ell,m}^{c,1} = -b_{\ell,m}^{e,1} \) for \( 1 \leq m \leq M \) in the proposed type II PTS scheme, as verified by using (20) as follows:

\[
\mathbf{x}_\ell^c = \left[ \sum_{m=1}^{M} b_{\ell,m}^{c,0} \mathbf{x}_{\ell,m} + \sum_{m=1}^{M} b_{\ell,m}^{c,1} \mathbf{x}_{\ell,m} \right] = \left[ \sum_{m=1}^{M} b_{\ell,m}^{c,0} \mathbf{x}_{\ell,m}^0 + \sum_{m=1}^{M} b_{\ell,m}^{c,1} \mathbf{x}_{\ell,m}^1 \right] \mathbf{x}_{\ell,m}^T \quad (25.a)
\]

\[
\mathbf{x}_\ell^e = \left[ \sum_{m=1}^{M} b_{\ell,m}^{e,0} \mathbf{x}_{\ell,m} + \sum_{m=1}^{M} b_{\ell,m}^{e,1} \mathbf{x}_{\ell,m} \right] = \left[ \sum_{m=1}^{M} b_{\ell,m}^{e,0} \mathbf{x}_{\ell,m}^0 + \sum_{m=1}^{M} b_{\ell,m}^{e,1} \mathbf{x}_{\ell,m}^1 \right] \mathbf{x}_{\ell,m}^T \quad (25.b)
\]

It is clear that \( \mathbf{x}_\ell^e \) of (25.b) is the cyclic-shift version of \( \mathbf{x}_\ell^c \) of (25.a), so \( \mathbf{x}_\ell^c \) and \( \mathbf{x}_\ell^e \) have the same PAPR value. These redundant candidate signals are removed in the optimization process of the proposed type II PTS scheme.

In addition, \( \mathbf{Q}_\ell^e \) of (21) and the similarity of the elements in \( \mathbf{Q}_\ell^c \) and \( \mathbf{x}_{\ell,m} \) in with \( 1 \leq i \leq 4 \) can also be used to reduce the computational complexity of the two proposed PTS schemes for SFBC MIMO-OFDM systems with \( N_T = 4 \) transmitting antennas, which is similar to the systems with \( N_T = 2 \) transmitting antennas described above. Given the similarity, we omit the discussion about simplifying the computational complexity of the proposed PTS schemes in SFBC MIMO-OFDM systems with \( N_T = 4 \).

V. SIMULATION RESULTS

In this section, computer simulations are used to compare the PAPR reduction performance, the BER performance, and the computational complexity of various PTS schemes in SFBC MIMO-OFDM systems. The system is assumed to have \( N = 256 \) subcarriers with a 16-QAM and 64-QAM constellation. The mean power \( \sigma^2 = 1 \) and the minimum possible peak power \( \Phi_{\mathbf{W}} = 4.204 \) with probability \( \gamma = 0.9999 \). During simulations, the \( L = 4 \) times oversampling technique is used to obtain the discrete-time signal. In the proposed type I PTS
scheme, the oPTS scheme, and the co-PTS scheme, the input data blocks are partitioned into $M_b = 8$ subblocks using the adjacent SPM for each transmitting antenna. In the proposed type II PTS scheme, the input data blocks are partitioned into $M_b = 8$ subblocks for each antenna using the methods described in Section III. During the computer simulation, the number of candidate signals is $C = 128$ for all PTS schemes. The rotation factor $b_{c,i}^e$ is fixed to 1; $b_{c,i}^e$ is selected from the set $\Theta_1 = \{1, -1\}$ for $1 \leq c \leq 64$ and from the set $\Theta_2 = \{1, j\}$ for $65 \leq c \leq 128$ to avoid redundant candidate signals.

A. Comparison of PAPR Reduction Performance

Figs. 4–6 show the complementary cumulative distribution function (CCDF) of the PAPR for various PTS scheme in SFBC MIMO-OFDM systems with $N_T = 2$ transmitting antennas. Fig. 4 shows the PAPR reduction performance for the oPTS, the co-PTS, and the proposed type I PTS scheme with different thresholds $\alpha_{TH} = \lambda \Phi_N^c$ where $\lambda$ = 0.125, 0.235, and 0.25. It shows that the proposed type I PTS scheme with $\lambda \leq 0.235$ achieves almost the same PAPR reduction performance as the oPTS scheme that uses all samples to compute the peak power.

Fig. 5 shows a comparison of PAPR reduction performance for the proposed type II PTS scheme with different thresholds $\alpha_{TH} = \lambda \Phi_N^e$ and the other two methods. It shows that the proposed type II PTS scheme with $\lambda \leq 0.24$ has almost the same PAPR reduction performance as the case that uses all samples for the peak power estimation. Compared with the oPTS and the co-PTS scheme, the proposed type II PTS scheme has 0.2 dB degradation in PAPR reduction performance when CCDF is $10^{-3}$. This is due to the fact that the first and second halves of elements of time-domain signals in (15) have correlation, which limits the randomness of candidate signals formed. Also, Fig. 6 shows the PAPR reduction performance of the four PTS schemes in SFBC MIMO-OFDM systems with a 64-QAM constellation. These results are almost the same as those shown in Figs. 4 and 5 for the systems with a 16-QAM constellation.

B. BER Performance

Fig. 7 shows a comparison of the PAPR reduction performance for the four PTS schemes in SFBC MIMO-OFDM systems with $N_T = 4$ and a 16-QAM constellation. Fig. 7 illustrates that the type I PTS scheme with $\alpha_{TH} = 0.25 \Phi_N^e$ and $p_\alpha = 0.40$ can achieve a PAPR reduction performance close to that of the oPTS scheme. The proposed type II PTS scheme with $\alpha_{TH} = 0.26 \Phi_N^e$ and $p_\alpha = 0.36$ shows a slight degradation in the PAPR reduction performance in comparison with the oPTS scheme.

B. BER Performance

Fig. 8 shows the BER performance of various PTS schemes in SFBC MIMO-OFDM systems with $N_T = 2$ transmitting antennas, where the AWGN channel is considered. The solid-state power amplifier (SSPA) model presented in [23] is used...
for simulation, in which the input back-off (IBO) is 3 dB and the parameter $p$ is 3. Assume that the side information is decoded correctly by the receiver for all PTS schemes. Fig. 8 shows that these PTS schemes improve the BER performance of the SFBC MIMO-OFDM systems. In comparison to the oPTS scheme, the proposed type I PTS scheme with $\alpha_{TH} = 0.235\Phi_N^\nu$ has almost the same BER performance; the proposed type II scheme with $\alpha_{TH} = 0.24\Phi_N^\nu$ shows a minor degradation in BER performance because of the higher PAPR value of the transmitted signals.

For simulation, in which the input back-off (IBO) is 3 dB and the parameter $p$ is 3. Assume that the side information is decoded correctly by the receiver for all PTS schemes. Fig. 8 shows that these PTS schemes improve the BER performance of the SFBC MIMO-OFDM systems. In comparison to the oPTS scheme, the proposed type I PTS scheme with $\alpha_{TH} = 0.235\Phi_N^\nu$ has almost the same BER performance; the proposed type II scheme with $\alpha_{TH} = 0.24\Phi_N^\nu$ shows a minor degradation in BER performance because of the higher PAPR value of the transmitted signals.

The proposed type I PTS scheme with $\alpha_{TH} = 0.26\Phi_N^\nu$ shows a slight degradation in BER performance, as compared with that of the oPTS scheme.

C. Comparison of Computational Complexity

Comparison of the computational complexity of various PTS schemes in SFBC MIMO-OFDM systems with $N_T = 2$ is discussed in this subsection. From Figs. 4 and 5, we note that the proposed type I and type II PTS schemes with $\lambda = 0.235$ have almost the same PAPR reduction performance as in the case that uses all samples to estimate the peak power of candidate signals, in which $\alpha_{TH} = 0.988$ and the corresponding $p_\alpha = 0.466$. Thus, the two proposed PTS schemes use only 46.6% of the samples to estimate the peak power of each candidate signal.

Assume that the computational complexity of the oPTS scheme is regarded as 100% and that the terms in Table I are multiplied by 2, the number of transmitting antennas, except for the terms with an asterisk. Then, we note that, in comparison to the oPTS scheme, the proposed type I PTS scheme requires only 52.9% complex multiplications, 47.1% real additions, and 46.6% multiplications of rotation factors. The proposed type II PTS scheme requires only 48.2% complex multiplications, 28.1% real additions, and 23.3% multiplications of rotation factors. The co-PTS scheme has the same complex multiplications and real additions as the oPTS scheme, but has only 1.8% multiplications of rotation factors, the least of the four methods. The proposed type II PTS scheme has the lowest computational complexity (except for the multiplications of rotation factors) of the four methods, at the cost of minor degradation in PAPR reduction performance and BER performance. Note that the multiplications of phase factors are usually ignored if the phase factors are selected from the set $\{\pm1, \pm j\}$. The numerical results of complexity are illustrated below the corresponding algebraic analyses in Table I.

In addition, Fig. 9 shows that the proposed type I and type II PTS schemes with $\alpha_{TH} = 0.26\Phi_N^\nu$ and $p_\alpha = 0.36$ have a good BER performance in SFBC MIMO-OFDM systems with $N_T = 4$, where 36% of samples are used for peak power estimation in each candidate signal. Thus, the two proposed
other two PTS schemes, similar to the systems with $N_s$ schemes also have lower computational complexity than the other two PTS schemes, similar to the systems with $N_T = 2$.

VI. Conclusion

In this paper, we have proposed two types of low-complexity PTS schemes to reduce the PAPR of the transmitted signals of SFBC MIMO-OFDM systems. The proposed type I PTS scheme uses the conventional PTS scheme in each transmitting antenna. The proposed type II PTS scheme uses the combination of interleaving and adjacent SPMs to partition the input data block into subblocks. Both proposed methods use the IFFT output of each subblock to generate cost functions for selecting samples, and only the selected samples are used to estimate the peak power of each candidate signal. In the proposed type II PTS scheme, we further use the correlation among subblocks to reduce the computational complexity for generating cost functions and candidate signals, as well as the IFFT computation. Simulation results show that, in SFBC MIMO-OFDM systems with two and four transmitting antennas, the proposed type I and type II PTS schemes can achieve a PAPR reduction performance and a BER performance close to those of the oPTS and the co-PTS schemes, but with much lower computational complexity.

REFERENCES


